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IDEAL BOOTSTRAP SURVIVAL FUNCTION AND ITS VARIANCE ESTIMATOR

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Abstract: Most of the data sets regarding survival analysis contain ties and censored events. The earlier methods do not take in to account the presence of these events and provide misleading results. In the present study, a method of Bootstrap survival function and its variance estimator by considering the ties among the events and censored observation is proposed. The performance of proposed method is checked by conducting simulations and compared with the existing methods. The results demonstrates that the proposed method perform better as compared to the earlier methods. Therefore, it is suggested that the proposed method be considered for the analysis of such types of data sets.

Key words: Bootstrap, Survival function, Variance estimator, Censored Observation.

Introduction

One of the main purposes of statistical methods is to estimate parameters, of a model, and to test their reliability. The idea of Bootstrapping, a resampling method, which is considered as one of fascinating discoveries in statistical methods was given by Efron (1979) for estimating the parameters with reliability. He has written much about the method of Bootstrap and its generalizations (Efron 1979, 1981).

In the past two decades thousands of papers have been published on Bootstrap method and their application in data analysis from a wide range of disciplines. The idea of bootstrap survival function for the estimation of the survival probabilities (Efron, 1981) is based on the restriction that there will be no ties between censored and uncensored observations in the data set. In case of any ties between censored and uncensored observations, the censored observation will be considered slightly larger than the uncensored observation. But in real situations, the data set contains tie cases between censored and uncensored events. If the ties observations are not considered in the analysis, it may seriously affect the estimates of various parameters and hence their associated tests (Zaman and Pfeiffer, 2005). Keeping in view the importance of ties and censored cases, an alternative method for estimating the Bootstrap survival function by considering the tie cases in the bootstrapping process, is proposed. In addition, a variance estimator for the proposed bootstrap survival function is also derived by applying the Delta method. The performance of proposed method as compared to the existing method is evaluated through extensive simulations. The proposed estimator is then applied to a real data set (Leukemia data set used Freireich et al. 1963). The results revealed that the proposed variance estimator perform better as compared to the existing method for estimating the survival function and recommended tom be used for data analysis.

Proposed Estimators

We proposed bootstrap survival function to overcome the problem in Efron (1981) by considering the tie cases in the bootstrapping process. The ties cannot be excluded from the data set as it affects the probabilities of survival badly, especially at the beginning of the study.

Let the survival times be $t_1, t_2 \ldots t_n$, where the t_j's are independently identically distributed, some of the t_j values may not be events and are censored. The data is of the form $\{(t_1, d_1), (t_2, d_2) \ldots (t_n, d_n)\}$, where t_j is the j^{th} observation, censored or not, and let the tie cases be considered by a binary variable and is defined as

$$\tau_j = \begin{cases} 1 & \text{if there is tie} \\ 0 & \text{otherwise} \end{cases}$$

Bootstrap samples are drawn, and the serial numbers are considered by $sn_1, sn_2, sn_3 \ldots$ and define $m_j^+ = No.\ of\ times(sn_j)$, appears in the bootstrap sample. Let

$$M_j = \sum_{j=1}^{n} m_j^+ \qquad j = 1, 2, \ldots, n$$

ties are considered in sampling while drawing the serial numbers, but has no role in the survival probabilities, so

$$m_j = m_j^+ - \tau_j .$$

Therefore, the proposed bootstrap survival function is given by

$$\hat{S}_P(t) = \prod_{j:j \le n} \left\{ 1 - \frac{m_j}{M_j} \right\}^{e_j}$$

Where e_j is the events in d_j, otherwise d_j's are censored. A variance estimator is derived for the proposed bootstrap survival function. The Delta method is used for the derivation of proposed variance estimator. The proposed bootstrap survival function is given as

$$\hat{S}_P(t) = \prod_{j:j \le k} \left\{ 1 - \frac{m_j}{M_j} \right\}$$

Let $p_j = \left(\frac{m_j}{M_j} \right)$ as $E(m_j) = M_j p_j$, by using Binomial and $q_j = \left(1 - \frac{m_j}{M_j} \right)$, therefore

$$\hat{S}_P(t) = \prod_{j:j \le k} (\hat{q}_j)$$

Applying log on both sides, we have

$$\log\left\{\hat{S}_P(t)\right\} = \log\left[\prod_{j:j \le k} (\hat{q}_j) \right]$$

The above can be written as

$$\log\left\{\hat{S}_P(t)\right\} = \sum_{j-1}^{k} \log \hat{q}_j$$

Applying variance on both sides

$$Var\left[\log\left\{\hat{S}_P(t)\right\}\right] = \sum_{j-1}^{k} Var\left[\log \hat{q}_j\right] \quad (A)$$

Now let

$$Var\left[\log \hat{q}_j\right] = \left(\frac{1}{q_j} \right)^2 Var(q_j)$$

The above result is achieved by applying Delta method as $Var\left[\phi(\hat{\theta})\right] \approx (\phi'(\theta))^2 Var(\phi(\theta))$. Hence $Var(q_j) = m_j p_j q_j$ by using Binomial (Peto, 1977), so

$$Var\left[\log \hat{q}_j\right] = \left(\frac{1}{q_j} \right)^2 (m_j p_j q_j)$$

$$Var\left[\log \hat{q}_j\right] = \left(\frac{m_j p_j}{q_j} \right)$$

Applying summation on both sides

$$\sum_{j=1}^{k} Var\left[\log \hat{q}_j\right] = \sum_{j=1}^{k} \left(\frac{m_j p_j}{q_j} \right)$$

Substituting in equation (A), we get

$$Var\left[\log\left\{\hat{S}_P(t)\right\}\right] = \sum_{j-1}^{k} Var\left[\log \hat{q}_j\right] \approx \sum_{j=1}^{k} \left(\frac{m_j p_j}{q_j} \right) \quad (B)$$

Again applying the Delta method;

$$Var\left[\hat{S}_P(t)\right] = Var\left[\exp\left\{\log \hat{S}_P(t)\right\}\right]$$

$$Var\left[\hat{S}_P(t)\right] = \left[\hat{S}_P(t)\right] Var\left[\log \hat{S}_P(t)\right]$$

Using the above equation (B), we get

$$Var\left[\hat{S}_M(t)\right] = \left[\hat{S}_M(t)\right] \sum_{j=1}^{k} \left(\frac{m_j p_j}{q_j}\right)$$

Now put $p_j = \left(\frac{m_j}{M_j}\right)$ and $q_j = \left(1 - \frac{m_j}{M_j}\right)$, we have finally the Proposed Bootstrap variance estimator, given by the expression

$$Var\left[\hat{S}_M(t)\right] = \left[\hat{S}_M(t)\right] \sum_{j=1}^{k} \left(\frac{m_j}{M_j(M_j - m_j)}\right)$$

The Bootstrap Estimator

Efron (1981) gave an idea of bootstrap survival function for the estimation of survival probabilities in survival analysis and its variance estimator. The bootstrap survival function is based on the assumption that there will be no tie between censored and ??? events (uncensored) observations. If it happens, the censored values are considered just slightly larger than the values actually recorded.

The data is of the form {(X_1, d_1), (X_2, d_2). . . (X_n, d_n)}, where X_i is the i^{th} observation, censored or not, is

$$d_i = \begin{cases} 1 & if\ X_i\ is\ uncensored \\ 0 & if\ X_i\ is\ censored \end{cases}$$

We will assume $X_1 < X_2 < \ldots < X_n$ are observed ordered survival times. Now (1) we draw a bootstrap sample (X_1^+, D_1^+), (X_2^+, D_2^+), . . ., (X_n^+, D_n^+) by independent sampling n times with replacement from $\hat{F}$, (2) letting + data represent this artificial data set, we calculate $\hat{\theta}^+ = \theta(^+data)$; (3) and independently repeat the steps (1) and (2) N times. The D_i^+ is defined as

$$D_i^+ = \begin{cases} 1 & if\ X_i^+\ is\ uncensored \\ 0 & if\ X_i^+\ is\ censored \end{cases}$$

Now considering drawing a single bootstrap sample (X_1^+, D_1^+), (X_2^+, D_2^+), . . ., (X_n^+, D_n^+) and define

$$m_i^* = \#times(X_i, d_i)$$

The above appears in the bootstrap sample, so $m^* = (m_1^*, m_2^* \ldots m_n^*)$ is an n-category multinomial n draws, with probability 1/n for each category: $m^* \sim$ multi(n, 1/n). Drawing "N" bootstraps, a bootstrap survival function can be found by applying Kaplan-Meier (1958) survival function to the bootstrap data set. But, before drawing the samples, we ignored the tied cases.

In turn Efron (1981) also suggested that variance estimator for the Bootstrap survival function. This variance estimator is quite similar to the usual derivation of Greenwood's (1926) variance estimator for grouped data.

Analysis

Estimators and its Analysis through Leukemia Clinical Trial

In this part of the study the estimators are applied to a real data set of Leukemia, conducted by Freireich et al. (1963). The data consists of n = 21 leukemia patients, treated with a new drug called 6-mercaptopurine. Out of these, patients 9 observed relapses, 12 were censored and at two time points are ties. The time (weeks) in remission are: 6, 6, 6, 6*, 7, 9*, 10, 10*, 11*, 13, 16, 17*, 19*, 20*, 22, 23, 25*, 32*, 32*, 34* and 35*, where '*' sign denotes a censored observation.

Before applying the survival function, we draw a bootstrap sample of size 50, 100, 200, 500 and 1000 with replacement from the given data set. After applying the bootstrap survival function and proposed bootstrap

survival function to the given data set, the results for survival functions are displayed in Table 1. Although the original data set is very small, still the proposed bootstrap method gives better results compared to its counterpart. While the results for small samples (for example; 50, 100 and 200) the improvement is more significant for proposed bootstrap survival function.

On the other hand, we also applied the Standard Errors of bootstrap and proposed bootstrap estimators to the same data set. With same sample data and survival functions, the results are displayed in Table 1. The results and the Figure 1, also suggests that the proposed bootstrap standard error is more significant than the bootstrap standard error.

Table 1: Standard Errors of Bootstrap and proposed estimators, applied to the real data set of Leukemia, with resample of size 500.

					Survival function		Standard Error	
Time	e_i	c_i	n_i	**tie**	$\hat{S}_B(t)$	$\hat{S}_M(t)$	SE_B	SE_{PB}
6	3	1	21	1	0.9280	0.9380	0.0107	0.0095
7	1	0	17	0	0.8766	0.8818	0.0150	0.0142
9	0	1	16	0	0.8766	0.8818	0.0150	0.0142
10	1	1	15	1	0.8216	0.8076	0.0181	0.0180
11	0	1	13	0	0.8216	0.8076	0.0181	0.0180
13	1	0	12	0	0.7233	0.7664	0.0211	0.0210
16	1	0	11	0	0.6332	0.7001	0.0228	0.0225
17	0	1	10	0	0.6332	0.7001	0.0228	0.0225
19	0	1	9	0	0.6332	0.7001	0.0228	0.0225
20	0	1	8	0	0.6332	0.7001	0.0228	0.0225
22	1	0	7	0	0.5008	0.5875	0.0262	0.0253
23	1	0	6	0	0.4014	0.4679	0.0271	0.0266
25	0	1	5	0	0.4014	0.4679	0.0271	0.0266
32	0	2	4	0	0.4014	0.4679	0.0271	0.0266
34	0	1	2	0	0.4014	0.4679	0.0271	0.0266
35	0	1	1	0	0.4014	0.4679	0.0271	0.0266

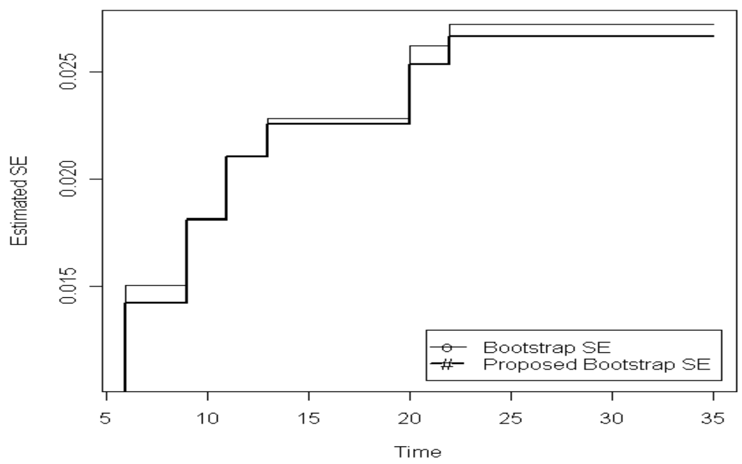

Fig. 1. The Standard Errors of Bootstrap and Proposed Bootstrap Estimators (The Estimators are applied to the Leukemia real data set).

Simulations

Simulation is one of the resampling techniques, widely used in statistics for comparing different estimators. Here the simulation method is used for comparison of proposed bootstrap survival function with the Bootstrap survival function. We also compared the proposed variance estimator with the Bootstrap variance.

For comparison, survival times are selected from Weibull, Exponential and Lognormal distributions. While for the censoring, a Uniform distribution is considered for selecting the survival times. The survival times and censoring times are then combined, to have a sample of sizes (n = 25, 50, 100, 500). Each data set of size n is then repeated 500 times.

For generating survival times the distributions are Exponential with parameter 0.5, 1.0 and 1.5. The Weibull distribution with first parameter fixed at one and the second parameter having values 0.5, 1.5. While the Lognormal distribution with first parameter fixed at one and for second parameter we considered 0.5, 1.0 and 1.5. On the other hand, for generating the censored times a Uniform distribution with density U (0, b) is used. Where b is adjusted such that it provides 15%, 25%, 24%, 65% and 85% censoring.

Survival Function Comparison

In this part the Bootstrap and proposed Bootstrap survival function are compared. First of all, for Bootstrapping, the sample sizes n are multiplied by five to have a resample of sizes (n = 125, 250, 500, 2500 respectively).

From each repetition of sample sizes, survival probabilities are calculated for Bootstrap and proposed bootstrap survival function. Three points are selected from each survival probabilities. They are the Quartiles (that is; q_1, q_2 and q_3, less than or equal to survival probabilities 0.25, 0.50 and 0.75 respectively) are stored. From each data set with resample of size n, repeated 500 times and we get 500 quartiles. Finally standard deviations are computed from these three Quartiles. The results are shown in Table 2(a), Table 2(b) and Table 2(c), with Figure 2. Seeing at these results, when the sample size n increases, the results of standard deviations for both estimators become smaller as expected. Also, when censoring increases the result for both the estimators decreases.

On comparing both the estimators at different quartiles, it was noticed that the proposed bootstrap survival function has much better performances as compared to the Bootstrap survival function. But in very few of the cases the Bootstrap survival function performs better.

At the 3rd quartile, the proposed Bootstrap survival function and Bootstrap survival function have almost same performances, compared to the 2nd quartile where proposed Bootstrap survival function performs better than the Bootstrap survival function. In case of 1st quartile the proposed Bootstrap survival function is more efficient compared to the Bootstrap survival function.

On the other hand, for different sample sizes it was observed that, as sample sizes become larger, the proposed Bootstrap survival function is more efficient than that of Bootstrap survival function.

Standard Errors Comparison

In next part of the simulations, we compared the Bootstrap and proposed Bootstrap standard errors. The means are calculated for the three quartiles achieved from these estimators. The results are shown in Table 3(a), 3(b), 3(c) and Figure 3. From

results, it is evident that when sample size *n* increases, the results for means of all the standard errors estimators become smaller as expected. By comparing the proposed Bootstrap standard error results with that of Bootstrap standard error, while applying Weibull distribution, the former shows better results. While in case of Log normal distribution, the results are almost the same for both the Bootstrap and proposed Bootstrap standard errors.

Here the results pattern are same for all the censoring levels except 25% and with the sample sizes n = 25, 50, 100 and 500. As censoring levels and sample sizes increases, the result for the means of quartiles decreases respectively.

Conclusions

The proposed estimators and their performances are discussed here in detail. The results of proposed estimators are also compared with the results of available estimators. For comparison purpose, the estimators are applied to the real data set by conducting extensive simulations.

The censoring is supposed to be non-informative, in calculating the survival function either used by Efron (1981) or by Kaplan-Meier (1958) method. Here we still suppose that the censoring is non-informative, but in reality occurring simultaneously with the event, and it carries some information. Here we just utilized the information of these ties in the Bootstrap resampling process by assigning “0” to no ties and “1” for tie between events and censored.

On the basis of these analysis (both simulations and real data analysis), it is concluded that the proposed estimators performs better in all the cases. The proposed estimators are very simple to calculate and provide better results than the existing estimators of survival functions. In addition, the results demonstrate that the tie cases play a significant role in the survival analysis and do affect the results. Therefore, it is suggested that the role of tied cases be considered so that best results can be achieved about the survival functions.

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Table 2(a): Standard Deviation (SD) of the Quartiles computed from the Bootstrap and proposed Bootstrap survival functions for different sample sizes and censoring percentages by using Uniform distribution for censoring and Weibull distribution for survival data.

Survivor	censored	Percent	n	Bootstrap Survival Function			Modified Bootstrap Survival Function		
				Standard Deviation			Standard Deviation		
				Q1	Q2	Q3	Q1	Q2	Q3
W(1,1.5)	U(0,10.2)	15	25	0.0249	0.0218	0.0192	0.0248	0.0218	0.0189
			50	0.0122	0.0103	0.0094	0.0121	0.0107	0.0099
			100	0.0061	0.0056	0.0052	0.0061	0.0050	0.0048
			500	0.0024	0.0018	0.0014	0.0020	0.0016	0.0015
W(1,1.5)	U(0,6.1)	25	25	0.0452	0.0233	0.0183	0.0470	0.0232	0.0201
			50	0.0189	0.0117	0.0102	0.0170	0.0116	0.0102
			100	0.0084	0.0059	0.0053	0.0077	0.0061	0.0049
			500	0.0024	0.0018	0.0015	0.0020	0.0017	0.0016
W(1,1.5)	U(0,2.9)	45	25	0.0894	0.0696	0.0240	0.0879	0.0540	0.0317
			50	0.0697	0.0216	0.0115	0.0673	0.0200	0.0119
			100	0.0313	0.0088	0.0059	0.0290	0.0085	0.0062
			500	0.0048	0.0024	0.0017	0.0031	0.0021	0.0017
W(1,1.5)	U(0,1.41)	65	25	0.0884	0.1812	0.0771	0.0888	0.1797	0.1010
			50	0.0988	0.1283	0.0199	0.0981	0.1247	0.0195
			100	0.0992	0.0471	0.0085	0.0949	0.0498	0.0084
			500	0.0223	0.0042	0.0023	0.0059	0.0032	0.0025
W(1,1.5)	U(0,0.52)	85	25	0.0403	0.1828	0.3112	0.0422	0.1793	0.3090
			50	0.0524	0.2011	0.2396	0.0501	0.1963	0.2362
			100	0.0621	0.2031	0.0579	0.0752	0.1920	0.0415
			500	0.0573	0.0091	0.0049	0.0104	0.0070	0.0059
W(1,0.5)	U(0,3.59)	15	25	0.0288	0.0224	0.0211	0.0276	0.0219	0.0181
			50	0.0142	0.0124	0.0109	0.0136	0.0111	0.0102
			100	0.0078	0.0069	0.0061	0.0069	0.0064	0.0060
			500	0.0033	0.0027	0.0024	0.0027	0.0025	0.0024
W(1,0.5)	U(0,2)	25	25	0.0484	0.0267	0.0203	0.0462	0.0262	0.0203
			50	0.0207	0.0137	0.0115	0.0164	0.0127	0.0111
			100	0.0145	0.0081	0.0063	0.0092	0.0075	0.0061
			500	0.0045	0.0031	0.0024	0.0031	0.0027	0.0027
W(1,0.5)	U(0,0.958)	45	25	0.0942	0.0711	0.0284	0.0908	0.0617	0.0280
			50	0.0743	0.0262	0.0123	0.0634	0.0228	0.0131
			100	0.0390	0.0106	0.0078	0.0358	0.0093	0.0076
			500	0.0102	0.0044	0.0029	0.0047	0.0037	0.0035
W(1,0.5)	U(0,0.48)	65	25	0.0918	0.1700	0.1083	0.0896	0.1668	0.0848
			50	0.0975	0.1213	0.0210	0.0979	0.0932	0.0199
			100	0.0994	0.0582	0.0102	0.0819	0.0276	0.0105
			500	0.0228	0.0071	0.0047	0.0070	0.0064	0.0062
W(1,0.5)	U(0,0.172)	85	25	0.0406	0.1903	0.2901	0.0420	0.1887	0.2766
			50	0.0634	0.2053	0.1518	0.0793	0.1939	0.1402
			100	0.0873	0.1105	0.0253	0.0977	0.0871	0.0252
			500	0.0514	0.0167	0.0121	0.0117	0.0204	0.0097

Table 2(b): Standard Deviation (SD) of the Quartiles computed from the Bootstrap and proposed Bootstrap survival functions for different sample sizes and censoring percentages by using Uniform distribution for censoring and Exponential distribution for survival data.

Survivor	censored	Percent	n	Bootstrap Survival Function			Modified Bootstrap Survival Function		
				Standard Deviation			Standard Deviation		
				Q_1	Q_2	Q_3	Q_1	Q_2	Q_3
E(1)	U(0,6.7)	15	25	0.0294	0.0235	0.0191	0.0257	0.0205	0.0191
			50	0.0138	0.0116	0.0099	0.0129	0.0111	0.0103
			100	0.0070	0.0061	0.0053	0.0066	0.0057	0.0052
			500	0.0023	0.0018	0.0016	0.0020	0.0017	0.0017
E(1)	U(0,4)	25	25	0.0450	0.0269	0.0206	0.0448	0.0253	0.0195
			50	0.0194	0.0121	0.0110	0.0181	0.0124	0.0103
			100	0.0093	0.0069	0.0054	0.0086	0.0062	0.0058
			500	0.0027	0.0021	0.0017	0.0023	0.0018	0.0018
E(1)	U(0,1.9)	45	25	0.0939	0.0646	0.0268	0.0912	0.0704	0.0263
			50	0.0657	0.0210	0.0124	0.0605	0.0207	0.0125
			100	0.0365	0.0098	0.0065	0.0336	0.0090	0.0066
			500	0.0059	0.0030	0.0019	0.0036	0.0024	0.0022
E(1)	U(0,0.95)	65	25	0.0889	0.1820	0.1236	0.0887	0.1846	0.0989
			50	0.1009	0.1235	0.0214	0.0999	0.1134	0.0217
			100	0.0987	0.0452	0.0090	0.0957	0.0211	0.0089
			500	0.0223	0.0047	0.0028	0.0054	0.0037	0.0033
E(1)	U(0,0.34)	85	25	0.0433	0.1925	0.2929	0.0536	0.1823	0.2958
			50	0.0611	0.2080	0.1971	0.0603	0.2044	0.2089
			100	0.0753	0.1776	0.0709	0.0940	0.1497	0.0611
			500	0.0483	0.0105	0.0063	0.0114	0.0089	0.0089
E(0.5)	U(0,13.4)	15	25	0.0273	0.0203	0.0197	0.0265	0.0198	0.0183
			50	0.0132	0.0106	0.0094	0.0120	0.0100	0.0093
			100	0.0060	0.0053	0.0051	0.0060	0.0050	0.0050
			500	0.0016	0.0014	0.0012	0.0014	0.0013	0.0012
E(0.5)	U(0,8)	25	25	0.0461	0.0237	0.0214	0.0422	0.0231	0.0188
			50	0.0205	0.0118	0.0097	0.0170	0.0116	0.0098
			100	0.0089	0.0059	0.0053	0.0078	0.0058	0.0052
			500	0.0020	0.0016	0.0013	0.0018	0.0016	0.0013
E(0.5)	U(0,3.8)	45	25	0.0950	0.0667	0.0276	0.0944	0.0658	0.0284
			50	0.0664	0.0201	0.0119	0.0646	0.0187	0.0117
			100	0.0311	0.0097	0.0059	0.0231	0.0086	0.0058
			500	0.0041	0.0022	0.0016	0.0033	0.0019	0.0015
E(0.5)	U(0,1.87)	65	25	0.0939	0.1801	0.0940	0.0906	0.1753	0.0887
			50	0.0986	0.1313	0.0232	0.0984	0.1191	0.0252
			100	0.1013	0.0679	0.0092	0.1000	0.0554	0.0088
			500	0.0336	0.0039	0.0020	0.0071	0.0027	0.0022
E(0.5)	U(0,0.68)	85	25	0.0318	0.1758	0.3146	0.0397	0.1729	0.3093
			50	0.0542	0.2011	0.2275	0.0529	0.1995	0.2294
			100	0.0682	0.2081	0.0978	0.0674	0.2074	0.0900
			500	0.0649	0.0090	0.0041	0.0219	0.0063	0.0049
E(1.5)	U(0,4.6)	15	25	0.0303	0.0216	0.0195	0.0259	0.0213	0.0197
			50	0.0136	0.0110	0.0108	0.0134	0.0117	0.0101
			100	0.0070	0.0066	0.0054	0.0069	0.0062	0.0058
			500	0.0028	0.0024	0.0019	0.0023	0.0022	0.0022
E(1.5)	U(0,2.69)	25	25	0.0555	0.0287	0.0198	0.0483	0.0243	0.0213
			50	0.0230	0.0121	0.0112	0.0202	0.0120	0.0107
			100	0.0100	0.0078	0.0061	0.0087	0.0068	0.0065
			500	0.0037	0.0026	0.0021	0.0027	0.0024	0.0023
E(1.5)	U(0,1.27)	45	25	0.0945	0.0741	0.0272	0.0904	0.0654	0.0260
			50	0.0694	0.0193	0.0128	0.0665	0.0185	0.0118
			100	0.0413	0.0098	0.0067	0.0282	0.0090	0.0067

			500	0.0078	0.0037	0.0026	0.0039	0.0029	0.0028
E(1.5)	U(0,0.64)	65	25	0.0847	0.1765	0.0913	0.0918	0.1732	0.0879
			50	0.0990	0.1072	0.0182	0.0979	0.0925	0.0216
			100	0.0975	0.0416	0.0100	0.0884	0.0275	0.0100
			500	0.0233	0.0056	0.0035	0.0060	0.0048	0.0048
E(1.5)	U(0,0.23)	85	25	0.0519	0.1887	0.2867	0.0468	0.1907	0.2866
			50	0.0646	0.2074	0.1837	0.0700	0.2022	0.1835
			100	0.0835	0.1524	0.0399	0.1000	0.1192	0.0208
			500	0.0428	0.0140	0.0090	0.0144	0.0139	0.0150

Table 2(c):Standard Deviation (SD) of the Quartiles computed from the Bootstrap and proposed bootstrap survival functions. For censoring, Uniform distribution is used. While for survival data, Log-Normal distribution is used.

Survivor	Censored	Percent	n	Bootstrap Survival Function			Modified Bootstrap Survival Function		
				Standard Deviation			Standard Deviation		
				Q1	Q2	Q3	Q1	Q2	Q3
Lnorm(1,1)	U(0,30.5)	15	25	0.0270	0.0193	0.0172	0.0248	0.0195	0.0170
			50	0.0124	0.0097	0.0100	0.0120	0.0100	0.0093
			100	0.0061	0.0050	0.0046	0.0057	0.0050	0.0045
			500	0.0014	0.0012	0.0010	0.0013	0.0011	0.0011
Lnorm(1,1)	U(0,16.8)	25	25	0.0437	0.0241	0.0188	0.0442	0.0235	0.0198
			50	0.0197	0.0114	0.0100	0.0226	0.0113	0.0095
			100	0.0076	0.0058	0.0049	0.0073	0.0058	0.0051
			500	0.0018	0.0013	0.0011	0.0016	0.0012	0.0011
Lnorm(1,1)	U(0,7.8)	45	25	0.0934	0.0709	0.0311	0.0919	0.0623	0.0243
			50	0.0648	0.0215	0.0113	0.0636	0.0208	0.0123
			100	0.0319	0.0093	0.0062	0.0353	0.0086	0.0060
			500	0.0034	0.0018	0.0013	0.0029	0.0017	0.0013
Lnorm(1,1)	U(0,4.09)	65	25	0.0852	0.1879	0.1184	0.0838	0.1877	0.0993
			50	0.0964	0.1376	0.0261	0.0988	0.1290	0.0257
			100	0.1016	0.0482	0.0085	0.1002	0.0455	0.0077
			500	0.0493	0.0036	0.0018	0.0346	0.0030	0.0017
Lnorm(1,1)	U(0,1.88)	85	25	0.0360	0.1726	0.3160	0.0351	0.1759	0.3160
			50	0.0505	0.1920	0.2604	0.0461	0.1914	0.2719
			100	0.0988	0.0488	0.0033	0.1031	0.0265	0.0032
			500	0.0602	0.2096	0.1688	0.0528	0.2092	0.1720
Lnorm(1,1.5)	U(0,46)	15	25	0.0290	0.0197	0.0179	0.0254	0.0194	0.0185
			50	0.0124	0.0100	0.0095	0.0121	0.0092	0.0091
			100	0.0062	0.0049	0.0048	0.0059	0.0048	0.0047
			500	0.0013	0.0012	0.0011	0.0012	0.0011	0.0010
Lnorm(1,1.5)	U(0,24)	25	25	0.0468	0.0240	0.0197	0.0453	0.0238	0.0205
			50	0.0160	0.0107	0.0099	0.0156	0.0107	0.0095
			100	0.0078	0.0059	0.0050	0.0068	0.0054	0.0048
			500	0.0016	0.0013	0.0012	0.0015	0.0012	0.0011
Lnorm(1,1.5)	U(0,8.7)	45	25	0.0901	0.0657	0.0236	0.0885	0.0621	0.0243
			50	0.0667	0.0195	0.0121	0.0685	0.0170	0.0104
			100	0.0305	0.0091	0.0060	0.0281	0.0083	0.0057
			500	0.0035	0.0018	0.0013	0.0029	0.0017	0.0013
Lnorm(1,1.5)	U(0,3.6)	65	25	0.0915	0.1782	0.0864	0.0868	0.1766	0.0853
			50	0.1005	0.1235	0.0384	0.0982	0.1196	0.0379
			100	0.0980	0.0341	0.0089	0.0957	0.0345	0.0086
			500	0.0402	0.0035	0.0018	0.0250	0.0029	0.0017
Lnorm(1,1.5)	U(0,1.27)	85	25	0.0416	0.1838	0.3115	0.0461	0.1769	0.3110
			50	0.0484	0.1991	0.2645	0.0528	0.1953	0.2615
			100	0.0588	0.2126	0.1240	0.0645	0.2115	0.1177
			500	0.1019	0.0118	0.0034	0.0794	0.0076	0.0034

Lnorm(1,0.5)	U(0,21.34)	15	25	0.0288	0.0218	0.0189	0.0288	0.0205	0.0192
			50	0.0124	0.0099	0.0093	0.0121	0.0095	0.0092
			100	0.0063	0.0052	0.0047	0.0058	0.0050	0.0047
			500	0.0016	0.0013	0.0012	0.0014	0.0012	0.0012
Lnorm(1,0.5)	U(0,12.7)	25	25	0.0476	0.0270	0.0192	0.0439	0.0235	0.0196
			50	0.0165	0.0115	0.0104	0.0160	0.0115	0.0099
			100	0.0079	0.0058	0.0052	0.0075	0.0058	0.0052
			500	0.0018	0.0015	0.0013	0.0016	0.0013	0.0012
Lnorm(1,0.5)	U(0,6.8)	45	25	0.0925	0.0697	0.0256	0.0899	0.0586	0.0243
			50	0.0689	0.0197	0.0115	0.0628	0.0171	0.0115
			100	0.0282	0.0088	0.0060	0.0236	0.0081	0.0057
			500	0.0038	0.0020	0.0013	0.0028	0.0017	0.0013
Lnorm(1,0.5)	U(0,4.38)	65	25	0.0896	0.1856	0.0954	0.0891	0.1772	0.1042
			50	0.0995	0.1214	0.0497	0.0976	0.1210	0.0372
			100	0.1017	0.0519	0.0081	0.0995	0.0379	0.0075
			500	0.0419	0.0033	0.0017	0.0211	0.0027	0.0017
Lnorm(1,0.5)	U(0,2.8)	85	25	0.0471	0.1757	0.3105	0.0405	0.1763	0.3088
			50	0.0518	0.2014	0.2642	0.0507	0.1987	0.2600
			100	0.0508	0.2123	0.1617	0.0548	0.2112	0.1464
			500	0.0973	0.0782	0.0034	0.0995	0.0579	0.0033

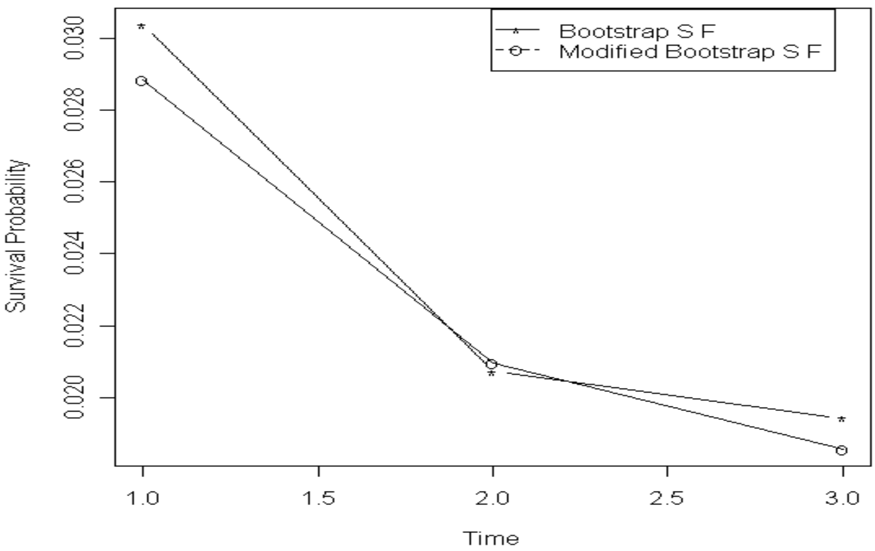

Fig. 2. The survival probability curves for Bootstrap, and proposed Bootstrap Survival Functions at three quartiles selected from the survival probabilities.

Table 3(a): Mean of the Quarters, computed from the Bootstrap and proposed Bootstrap Standard Errors. For censoring, Uniform distribution is used. While for survival data, Weibull distribution is used.

Survivor	censored	Percent	n	Bootstrap SE			Proposed Bootstrap SE		
					MEAN			MEAN	
				Q1	Q2	Q3	Q1	Q2	Q3
W(1,1.5)	U(0,10.14)	15	25	0.0331	0.0408	0.0448	0.0318	0.0401	0.0447
			50	0.0230	0.0288	0.0317	0.0225	0.0286	0.0317
			100	0.0162	0.0204	0.0225	0.0160	0.0203	0.0224
			500	0.0073	0.0091	0.0101	0.0071	0.0090	0.0100
W(1,1.5)	U(0,6.1)	25	25	0.0340	0.0422	0.0465	0.0328	0.0417	0.0463
			50	0.0238	0.0299	0.0329	0.0235	0.0297	0.0328
			100	0.0168	0.0212	0.0233	0.0166	0.0210	0.0232
			500	0.0077	0.0095	0.0104	0.0074	0.0094	0.0103
W(1,1.5)	U(0,2.9)	45	25	0.0329	0.0435	0.0504	0.0317	0.0431	0.0502
			50	0.0231	0.0313	0.0363	0.0227	0.0310	0.0361
			100	0.0164	0.0223	0.0258	0.0163	0.0223	0.0257
			500	0.0071	0.0098	0.0116	0.0075	0.0101	0.0115
W(1,1.5)	U(0,1.43)	65	25	0.0268	0.0398	0.0528	0.0261	0.0396	0.0527
			50	0.0195	0.0290	0.0386	0.0193	0.0290	0.0386
			100	0.0137	0.0204	0.0276	0.0137	0.0206	0.0278
			500	0.0060	0.0089	0.0117	0.0065	0.0095	0.0125
W(1,1.5)	U(0,0.52)	85	25	0.0158	0.0281	0.0436	0.0157	0.0287	0.0437
			50	0.0121	0.0209	0.0323	0.0121	0.0211	0.0326
			100	0.0095	0.0156	0.0237	0.0095	0.0158	0.0242
			500	0.0046	0.0071	0.0101	0.0051	0.0079	0.0117
W(1,0.5)	U(0,3.58)	15	25	0.0333	0.0408	0.0447	0.0320	0.0403	0.0446
			50	0.0231	0.0289	0.0318	0.0226	0.0287	0.0317
			100	0.0162	0.0204	0.0225	0.0158	0.0202	0.0223
			500	0.0076	0.0093	0.0101	0.0071	0.0090	0.0099
W(1,0.5)	U(0,2)	25	25	0.0341	0.0422	0.0466	0.0330	0.0418	0.0464
			50	0.0238	0.0299	0.0330	0.0234	0.0297	0.0329
			100	0.0170	0.0213	0.0234	0.0167	0.0211	0.0232
			500	0.0076	0.0098	0.0106	0.0074	0.0093	0.0102
W(1,0.5)	U(0,0.95)	45	25	0.0322	0.0436	0.0505	0.0311	0.0431	0.0504
			50	0.0231	0.0312	0.0365	0.0229	0.0311	0.0363
			100	0.0161	0.0222	0.0260	0.0165	0.0223	0.0259
			500	0.0067	0.0093	0.0112	0.0077	0.0101	0.0112
W(1,0.5)	U(0,0.48)	65	25	0.0271	0.0403	0.0529	0.0266	0.0401	0.0527
			50	0.0194	0.0287	0.0385	0.0196	0.0291	0.0389
			100	0.0138	0.0204	0.0269	0.0142	0.0209	0.0276
			500	0.0058	0.0084	0.0106	0.0071	0.0098	0.0121
W(1,0.5)	U(0,0.172)	85	25	0.0164	0.0278	0.0429	0.0165	0.0279	0.0434
			50	0.0131	0.0213	0.0324	0.0133	0.0216	0.0333
			100	0.0099	0.0156	0.0229	0.0104	0.0167	0.0249
			500	0.0050	0.0073	0.0095	0.0065	0.0094	0.0123

Table 3(b):Mean of the Quarters, computed from the Bootstrap and proposed Bootstrap Standard Errors. For censoring, Uniform distribution is used. While for survival data, Exponential distribution is used.

Survivor	censored	Percent	n	Bootstrap SE			Proposed Bootstrap SE		
					MEAN			MEAN	
				Q1	Q2	Q3	Q1	Q2	Q3
E(1)	U(0,6.75)	15	25	0.0332	0.0409	0.0449	0.0319	0.0404	0.0448
			50	0.0230	0.0289	0.0318	0.0225	0.0287	0.0317
			100	0.0163	0.0204	0.0225	0.0160	0.0203	0.0224
			500	0.0074	0.0092	0.0101	0.0071	0.0090	0.0100
E(1)	U(0,4)	25	25	0.0342	0.0422	0.0462	0.0332	0.0418	0.0460
			50	0.0238	0.0299	0.0329	0.0234	0.0297	0.0327
			100	0.0170	0.0212	0.0233	0.0167	0.0211	0.0232
			500	0.0077	0.0096	0.0105	0.0075	0.0094	0.0103
E(1)	U(0,1.9)	45	25	0.0319	0.0433	0.0508	0.0314	0.0430	0.0505
			50	0.0232	0.0312	0.0365	0.0229	0.0310	0.0363
			100	0.0163	0.0223	0.0260	0.0163	0.0223	0.0259
			500	0.0069	0.0097	0.0115	0.0075	0.0101	0.0115
E(1)	U(0,0.95)	65	25	0.0273	0.0398	0.0521	0.0268	0.0400	0.0521
			50	0.0193	0.0288	0.0385	0.0192	0.0288	0.0386
			100	0.0135	0.0205	0.0274	0.0137	0.0207	0.0278
			500	0.0059	0.0087	0.0113	0.0066	0.0096	0.0124
E(1)	U(0,0.342)	85	25	0.0149	0.0268	0.0431	0.0147	0.0270	0.0433
			50	0.0125	0.0216	0.0323	0.0124	0.0217	0.0332
			100	0.0098	0.0156	0.0236	0.0099	0.0160	0.0245
			500	0.0047	0.0072	0.0098	0.0055	0.0084	0.0119
E(0.5)	U(0,13.5)	15	25	0.0331	0.0409	0.0448	0.0318	0.0404	0.0447
			50	0.0230	0.0288	0.0317	0.0226	0.0287	0.0317
			100	0.0162	0.0204	0.0225	0.0161	0.0203	0.0224
			500	0.0072	0.0091	0.0101	0.0071	0.0090	0.0100
E(0.5)	U(0,8)	25	25	0.0340	0.0423	0.0464	0.0329	0.0418	0.0463
			50	0.0237	0.0299	0.0329	0.0234	0.0297	0.0329
			100	0.0168	0.0212	0.0233	0.0167	0.0211	0.0233
			500	0.0076	0.0095	0.0104	0.0074	0.0094	0.0103
E(0.5)	U(0,3.8)	45	25	0.0322	0.0434	0.0507	0.0326	0.0434	0.0507
			50	0.0229	0.0312	0.0365	0.0229	0.0313	0.0365
			100	0.0162	0.0223	0.0261	0.0163	0.0223	0.0260
			500	0.0071	0.0099	0.0117	0.0074	0.0101	0.0116
E(0.5)	U(0,1.88)	65	25	0.0269	0.0402	0.0529	0.0270	0.0402	0.0530
			50	0.0193	0.0289	0.0385	0.0194	0.0290	0.0385
			100	0.0136	0.0206	0.0277	0.0137	0.0207	0.0277
			500	0.0066	0.0090	0.0119	0.0064	0.0094	0.0125
E(0.5)	U(0,0.68)	85	25	0.0155	0.0273	0.0422	0.0154	0.0273	0.0425
			50	0.0125	0.0214	0.0330	0.0126	0.0217	0.0333
			100	0.0095	0.0155	0.0234	0.0095	0.0157	0.0239
			500	0.0045	0.0071	0.0102	0.0048	0.0077	0.0115
E(1.5)	U(0,4.6)	15	25	0.0332	0.0408	0.0449	0.0332	0.0408	0.0449
			50	0.0232	0.0290	0.0319	0.0232	0.0289	0.0318
			100	0.0162	0.0204	0.0225	0.0162	0.0204	0.0224
			500	0.0075	0.0092	0.0101	0.0071	0.0090	0.0099
E(1.5)	U(0,2.69)	25	25	0.0336	0.0421	0.0464	0.0338	0.0421	0.0463
			50	0.0239	0.0299	0.0329	0.0238	0.0298	0.0329
			100	0.0169	0.0212	0.0233	0.0168	0.0211	0.0232
			500	0.0077	0.0097	0.0105	0.0075	0.0093	0.0102
E(1.5)	U(0,1.27)	45	25	0.0328	0.0436	0.0504	0.0327	0.0437	0.0505
			50	0.0229	0.0312	0.0365	0.0229	0.0313	0.0365
			100	0.0162	0.0222	0.0260	0.0164	0.0223	0.0258
			500	0.0068	0.0095	0.0114	0.0076	0.0101	0.0113

E(1.5)	U(0,0.64)	65	25	0.0272	0.0404	0.0527	0.0271	0.0403	0.0528
			50	0.0195	0.0289	0.0384	0.0197	0.0292	0.0387
			100	0.0137	0.0203	0.0271	0.0141	0.0208	0.0278
			500	0.0059	0.0085	0.0109	0.0069	0.0097	0.0123
E(1.5)	U(0,0.23)	85	25	0.0164	0.0291	0.0431	0.0167	0.0293	0.0436
			50	0.0131	0.0217	0.0324	0.0134	0.0222	0.0334
			100	0.0096	0.0156	0.0231	0.0099	0.0163	0.0248
			500	0.0049	0.0073	0.0096	0.0060	0.0089	0.0123

Table 3(c): Mean of the Quarters, computed from the Bootstrap and proposed Bootstrap Standard Errors. For censoring, Uniform distribution is used. While for survival data, Log-Normal distribution is used.

Survivor	Censored	Percent	n	Bootstrap SE			Proposed Bootstrap SE		
					MEAN			MEAN	
				Q1	Q2	Q3	Q1	Q2	Q3
Lnorm(1,1)	U(0,30.5)	15	25	0.0328	0.0406	0.0448	0.0329	0.0407	0.0448
			50	0.0230	0.0288	0.0317	0.0230	0.0288	0.0317
			100	0.0161	0.0203	0.0224	0.0161	0.0203	0.0224
			500	0.0072	0.0091	0.0100	0.0071	0.0091	0.0100
Lnorm(1,1)	U(0,16.8)	25	25	0.0340	0.0421	0.0466	0.0339	0.0420	0.0465
			50	0.0239	0.0300	0.0330	0.0239	0.0300	0.0330
			100	0.0168	0.0212	0.0233	0.0168	0.0212	0.0233
			500	0.0075	0.0095	0.0104	0.0075	0.0094	0.0104
Lnorm(1,1)	U(0,7.8)	45	25	0.0326	0.0433	0.0506	0.0327	0.0434	0.0506
			50	0.0233	0.0313	0.0365	0.0233	0.0313	0.0365
			100	0.0163	0.0222	0.0260	0.0163	0.0222	0.0260
			500	0.0072	0.0100	0.0118	0.0073	0.0101	0.0117
Lnorm(1,1)	U(0,4.09)	65	25	0.0270	0.0406	0.0531	0.0271	0.0402	0.0531
			50	0.0194	0.0292	0.0388	0.0197	0.0293	0.0388
			100	0.0139	0.0207	0.0278	0.0139	0.0207	0.0279
			500	0.0061	0.0092	0.0122	0.0063	0.0094	0.0125
Lnorm(1,1)	U(0,1.89)	85	25	0.0167	0.0281	0.0430	0.0167	0.0278	0.0423
			50	0.0123	0.0216	0.0325	0.0122	0.0216	0.0324
			100	0.0094	0.0154	0.0239	0.0093	0.0154	0.0242
			500	0.0044	0.0070	0.0106	0.0044	0.0072	0.0111
Lnorm(1,1.5)	U(0,47)	15	25	0.0333	0.0408	0.0449	0.0332	0.0408	0.0448
			50	0.0230	0.0288	0.0317	0.0230	0.0288	0.0317
			100	0.0162	0.0204	0.0225	0.0161	0.0204	0.0224
			500	0.0072	0.0091	0.0100	0.0071	0.0091	0.0100
Lnorm(1,1.5)	U(0,23.8)	25	25	0.0343	0.0422	0.0464	0.0340	0.0422	0.0465
			50	0.0239	0.0299	0.0329	0.0240	0.0299	0.0328
			100	0.0168	0.0211	0.0233	0.0168	0.0211	0.0233
			500	0.0075	0.0095	0.0104	0.0075	0.0094	0.0103
Lnorm(1,1.5)	U(0,8.7)	45	25	0.0328	0.0435	0.0507	0.0327	0.0434	0.0506
			50	0.0233	0.0313	0.0364	0.0233	0.0313	0.0363
			100	0.0163	0.0223	0.0260	0.0163	0.0223	0.0260
			500	0.0072	0.0100	0.0118	0.0073	0.0100	0.0117
Lnorm(1,1.5)	U(0,3.6)	65	25	0.0266	0.0398	0.0532	0.0269	0.0403	0.0531
			50	0.0195	0.0291	0.0387	0.0195	0.0293	0.0390
			100	0.0138	0.0205	0.0277	0.0140	0.0206	0.0277
			500	0.0061	0.0091	0.0122	0.0063	0.0093	0.0125
Lnorm(1,1.5)	U(0,1.27)	85	25	0.0165	0.0282	0.0433	0.0165	0.0280	0.0435
			50	0.0122	0.0215	0.0334	0.0123	0.0218	0.0335
			100	0.0097	0.0159	0.0239	0.0098	0.0159	0.0241
			500	0.0045	0.0071	0.0106	0.0046	0.0074	0.0113
Lnorm(1,0.5)	U(0,21.35)	15	25	0.0330	0.0408	0.0450	0.0331	0.0407	0.0448

			50	0.0230	0.0288	0.0317	0.0231	0.0289	0.0318
			100	0.0161	0.0204	0.0224	0.0161	0.0203	0.0224
			500	0.0072	0.0091	0.0100	0.0071	0.0091	0.0100
Lnorm(1,0.5)	U(0,12.68)	25	25	0.0340	0.0423	0.0464	0.0342	0.0422	0.0464
			50	0.0239	0.0299	0.0330	0.0238	0.0299	0.0329
			100	0.0168	0.0211	0.0233	0.0168	0.0211	0.0232
			500	0.0076	0.0095	0.0104	0.0074	0.0094	0.0103
Lnorm(1,0.5)	U(0,6.8)	45	25	0.0323	0.0436	0.0508	0.0322	0.0435	0.0506
			50	0.0234	0.0314	0.0364	0.0232	0.0312	0.0363
			100	0.0163	0.0223	0.0260	0.0164	0.0224	0.0260
			500	0.0072	0.0100	0.0117	0.0073	0.0100	0.0117
Lnorm(1,0.5)	U(0,4.38)	65	25	0.0268	0.0401	0.0534	0.0270	0.0402	0.0535
			50	0.0196	0.0291	0.0385	0.0196	0.0290	0.0386
			100	0.0137	0.0206	0.0278	0.0138	0.0207	0.0278
			500	0.0061	0.0091	0.0123	0.0062	0.0093	0.0125
Lnorm(1,0.5)	U(0,2.78)	85	25	0.0164	0.0273	0.0431	0.0165	0.0276	0.0429
			50	0.0131	0.0215	0.0329	0.0130	0.0216	0.0330
			100	0.0096	0.0157	0.0240	0.0096	0.0156	0.0242
			500	0.0044	0.0071	0.0108	0.0044	0.0072	0.0111

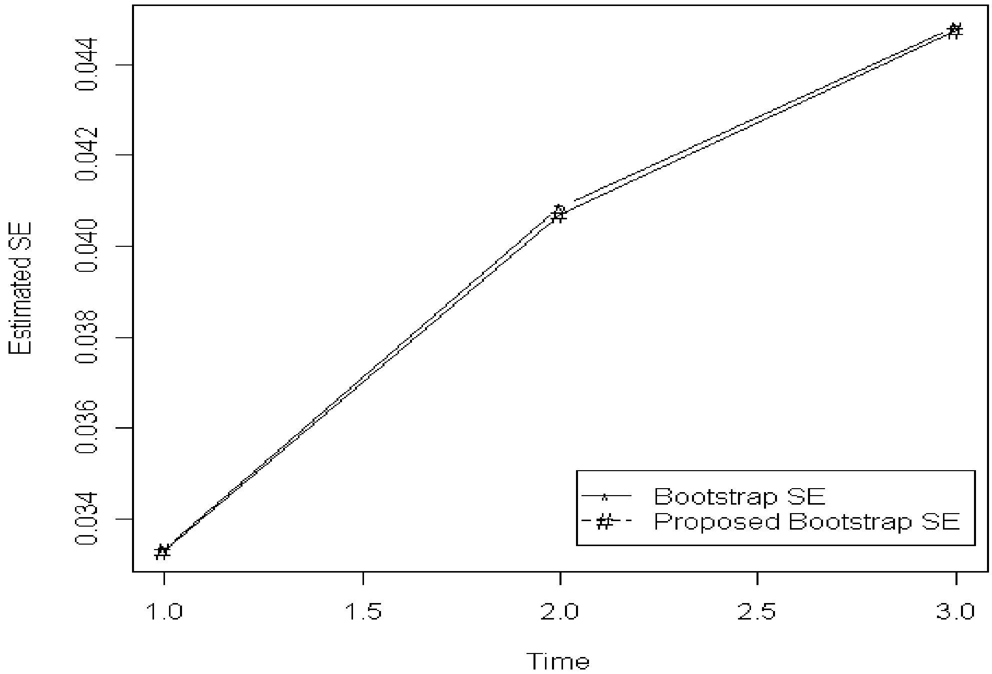

Fig. 3. The survival probability curves for Bootstrap and Proposed Bootstrap Standard Error at three quartiles selected from the survival probabilities